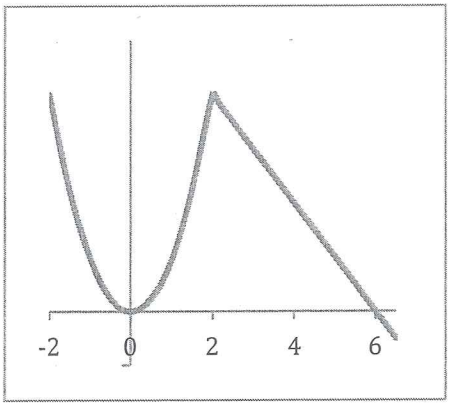
Sec 2.3 Piecewise Defined Functions

Example

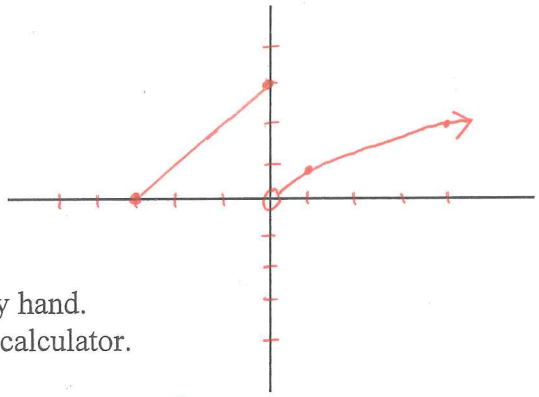
A function may employ different formulas on different parts of its domain. Such a function is said to be *piecewise defined.* For example, the function graphed has the following formulas:



$$y = \begin{cases} x^2 & \text{for } x \le 2 \\ 6 - x & \text{for } x > 2 \end{cases}$$

Piecewise-defined Functions – a function that is defined by more than one equation for different parts of the domain

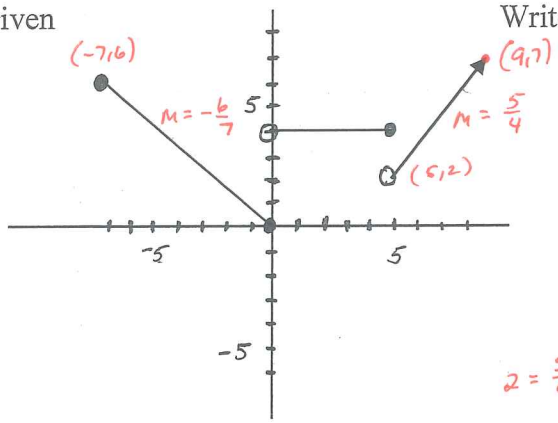
Ex. $f(x) = \begin{cases} 3 + x & \text{if } -3 \le x < 0 \\ 3 & \text{if } x = 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$



a. Find the domain.
a. Graph each function by hand.
b. Verify using graphing calculator.
c. Locate any intercepts.
d. Based on the graph, find the range. $f(x) \ge 1$
e. Find f(-2), f(0) and f(3).

1 3 $\sqrt{3} = 1.73$

Ex. Given



Write the definition for each function.

$$f(x) = \begin{cases} -\frac{6}{7}x & \text{if } -7 \le x \le 0 \\ 4 & \text{if } 0 < x \le 5 \\ \frac{5}{4}x - \frac{17}{4} & \text{if } x > 5 \end{cases}$$

$2 = \frac{5}{4} \cdot 5 + b$

$2 = \frac{25}{4} + b$

$\frac{8}{4} - \frac{25}{4} = b$

$-\frac{17}{4} = b$

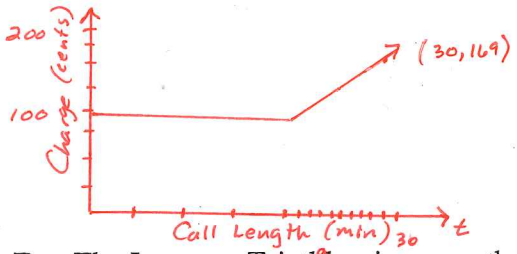
Ex. A long distance calling plan charges 99 cents for any call up to 20 minutes and 7 cents for each additional minute or part of a minute.

a. Use bracket notation to write a formula for the cost, C, of a call as a function of its length t in minutes.
b. Graph the function.
c. State the domain and range of the function.

$C = \begin{cases} 99 & \text{if } 0 < t \le 20 \\ 99 + 7(t-20) & \text{if } t > 20 \end{cases}$

$C = \begin{cases} 99 & \text{if } 0 < t \le 20 \\ 7t - 41 & \text{if } t > 20 \end{cases}$

$t > 0$ $C \ge 99$

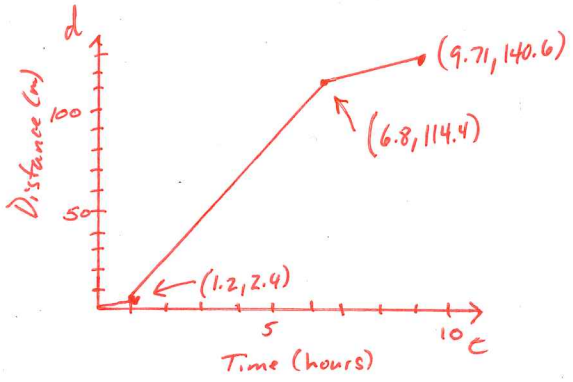


Ex. The Ironman Triathlon is a race that consists of three parts, a 2.4 mile swim followed by a 112 mile bike race, and then a 26.2 mile marathon. A participant swims steadily at 2 mph, cycles steadily at 20 mph, and then runs steadily at 9 mph. Assuming that no time is lost during the transition from one stage to the next, find a formula for the distance covered, *d*, in miles as a function of the elapsed time *t* in hours from the beginning of the race. Graph the function.

$2.4 + 20t - 24$
$20t - 21.6$
$114.4 + 9t - 61.2$
$9t + 53.2$

$d = \begin{cases} 2t & 0 \le t \le 1.2 \\ 2.4 + 20(t-1.2) & 1.2 < t \le 6.8 \\ 114.4 + 9(t-6.8) & 6.8 < t \le 9.71 \end{cases}$

$d = \begin{cases} 2t & \text{if } 0 \le t \le 1.2 \\ 20t - 21.6 & \text{if } 1.2 < t \le 6.8 \\ 9t + 53.2 & \text{if } 6.8 < t \le 9.71 \end{cases}$



Absolute Value Function –

i. Domain is the set of all real numbers and range is the set of all nonnegative real numbers
ii. Intercept of the graph is (0. 0)
iii. Even function decreasing on $(-\infty, 0)$ & increases from $(0, \infty)$
iv. F(x) = $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

HW: pg 83-86 #3, 6, 9, 10, 12, 14, 16, 18, 19, 22-24